# A Discrete-Time Multivariable Model-Following Method Applied to Decoupled Flight Control

Kimio Kanai\*
National Defense Academy, Yokosuka, Japan
and
Noriyuki Hori† and Peter N. Nikiforuk‡
University of Saskatchewan, Saskatoon, Canada

The discrete-time version of the multivariable model-following control method, previously presented by the authors of this paper,<sup>3</sup> is described. The modification of the control law that enables the control vector to be calculated at each instant using the past state vectors is also presented for synchronizing the change in the control input with the sampling instant. An alternative method of control is developed for a system whose high-frequency gain matrix is singular. This design requires a lower-order increase in the controller and makes the selection of the reference model easier than the method previously proposed by the authors. As an example, the design of the control system is given for flight control of an aircraft that has control-configured vehicle capabilities. The simulation study is carried out for a hypothetical T-2-CCV aircraft, with particular reference to a longitudinal mode.

## Introduction

DURING the past decade, the concept of the control-configured vehicle (CCV) has been developed to improve the flight performance of aircraft by directly controlling the vertical and horizontal forces that allow the linear and angular motions to be separated for various missions.<sup>1,2</sup> We have shown that this may be achieved by using a model-following controller so that, by selecting suitable inputs, outputs, and state variables, the plant may be decoupled and suitably compensated.<sup>3</sup> This controller, which is composed of input dynamics and state feedback blocks, makes the transfer matrix of the plant-controller combination equal to that of the desired model and, therefore, includes as a special case the decoupling of the system.

System decoupling using state feedback requires that a highfrequency gain matrix, which depends on the system's parameters, be nonsingular.4 We presented a method for designing a model-following controller in continuous-time form which overcomes this singularity problem, and carried out simulation studies for the longitudinal motion of CCVtype aircraft showing that this method is effective.3 The key to overcoming this singularity problem was that of postaugmenting the system using a unimodular polynomial matrix, § to assure the nonsingularity of a high-frequency gain matrix. As this method requires that the model as well as the plant be augmented, the order of the controller is increased considerably. An alternate method of system augmentation, which requires a smaller increase in the order, is developed in this paper by preaugmenting the plant, using a unimodular polynomial matrix. In addition, the selection of a reference model which satisfies the required row-relative degree condition is easier.

Since modern aircraft are equipped with digital computers, the controller should be designed in discrete-time form. In this study, a discrete-time model-following control method is presented, including a useful modification of the control law. If the continuous-time control law is directly discretized, the resulting law requires measurement at a time k of the state variables for the purpose of computing the control input that should be applied to the plant at the same instant, k. However, when a sophisticated control law is desired and a high sampling frequency is used, the computation time needed to execute its algorithm by digital computer may be significant, relative to the sampling interval. In this case, the control law needs to be modified by synchronizing the change of the control input with the sampling instant, to ensure that the input calculation at each instant is completed.

The application of the control scheme to the flight control of an aircraft that has CCV capabilities is considered in this paper, with particular reference to the longitudinal CCV modes, using simulation studies to substantiate the analytical work.

## **Model-Following Controller Synthesis**

With Nonsingular G: Case 1.

Consider a plant which is reachable, but may be unstable, and described by the following equations

$$x(k+1) = Ax(k) + Bu(k), \quad x(0) = x_0$$
 (1)

$$y(k) = Cx(k) \tag{2}$$

where  $x(k) \in \mathbb{R}^n$ , u(k), and  $y(k) \in \mathbb{R}^r$ . Its pulse transfer matrix is given by

$$H(z) = C(zI - A)^{-1}B$$
 (3)

where the letter z denotes the Z-transform or forward shift operator, depending upon the context. It is assumed that the rank of H(z) is r (full rank) and all of the plant zeros are located inside the unit circle in the z-plane.

A reference model which has the desired characteristics is represented by

$$x_M(k+1) = A_M x_M(k) + B_M u_M(k), \quad x_M(0) = x_{M_0}$$
 (4)

$$y_M(k) = C_M x_M(k) \tag{5}$$

Received July 1, 1985, presented as Paper 85-1978 at the AIAA Guidance, Navigation, and Control Conference, Snowmass, CO, Aug. 19-21, 1985; revision received Nov. 4, 1985. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1986. All rights reserved.

<sup>\*</sup>Professor of Aeronautical Engineering. Member AIAA.

<sup>†</sup>Postdoctoral Fellow, Mechanical Engineering. Member AIAA.

<sup>‡</sup>Dean of Engineering.

<sup>§</sup>A polynomial matrix whose determinant is a nonzero scalar is called unimodular; that is, its inverse is also a polynomial matrix.<sup>5</sup>

where  $x_M(k) \in R^n$ ,  $u_M(k)$  and  $y_M(k) \in R^r$ . The corresponding pulse transfer matrix is

$$H_M(z) = C_M(zI - A_M)^{-1}B_M$$
 (6)

where  $\det(zI - A_M)$  is a stable polynomial.

To avoid making a controller non-causal, the model should be selected such that each row-relative degree of its pulse transfer matrix is larger than, or equal to, the corresponding degree of the plant, or equivalently that

$$\bar{n} - \partial_r [C_M \operatorname{adj} (zI - A_M) B_M] \ge n - \partial_r [C \operatorname{adj} (zI - A) B]$$
 (7)

where  $\partial$ , denotes row degree.

Selecting the coefficients  $f_i^j(i=1,2,...,m_j, j=1,2,...,r)$ , where  $m_j$  is the jth row-relative degree of the plant pulse transfer matrix, such that the polynomial

$$f^{j}(z) = z^{m_{j}} + f_{1}^{j} z^{m_{j}-1} + \dots + f_{m_{j}}^{j}$$
 (8)

is stable, the following output error equation is obtained<sup>3</sup>

$$\Phi(z)e(k) = Fx(k) + Gu(k) - F_M x_M(k) - G_M u_M(k)$$
 (9)

where

$$\Phi(z) = \operatorname{diag}(f^{j}(z))$$

$$e_{j}(k) = y_{j}(k) - y_{M_{j}}(k), \quad e^{T}(k) = [e_{1}(k), e_{2}(k), ..., e_{r}(k)]$$

$$F = [F_{j}], \quad F_{j} = C_{j}A^{m_{j}} + f_{1}^{j}C_{j}A^{m_{j}-1} + ... + f_{m_{j}}^{j}C_{j}$$

$$G = [G_{j}], \quad G_{j} = C_{j}A^{m_{j}-1}B$$

$$F_{M} = [F_{M_{j}}], \quad F_{M_{j}} = C_{M_{j}}A_{M}^{m_{j}} + f_{1}^{j}C_{M_{j}}A_{M}^{m_{j}-1} + ...$$

$$+ f_{m_{j}}^{j}C_{M_{j}}(j = 1, 2, ..., r)$$

$$G_{M} = [G_{M_{j}}], \quad G_{M_{j}} = C_{M_{j}}A_{M}^{m_{j}-1}B_{M}$$

$$(10)$$

Define a control input such that the right-hand side of Eq. (9) is equal to zero; that is

$$u(k) = G^{-1} [-Fx(k) + q(k)]$$

$$q(k) = F_M x_M(k) + G_M u_M(k)$$
(11)

It can be shown<sup>3</sup> that the pulse transfer matrix of the feedback portion is  $\Phi^{-1}(z)$  and that of the feedforward path  $\Phi(z)H_M(z)$ . This makes the overall system have the desired transfer matrix  $H_M(z)$ . Figure 1 is a block diagram of this control system.

The calculation of the control input at the instant k using Eq. (11) requires the state variables at k. Using Eqs. (1) and (4), the input can be rewritten as

$$u(k) = G^{-1} [-Ex(k-1) - Nu(k-1) + q(k)]$$

$$q(k) = E_M x_M(k-1) + N_M u_M(k-1) + G_M u_M(k)$$
(12)

where

$$E=FA$$
,  $N=FB$ ,  $E_M=F_MA_M$ ,  $N_M=F_MB_M$  (13)

With this modification, the control u(k) can be applied as soon as the command input  $u_M(k)$  is applied to the controller. This is particularly important when the on-board computer has a processing speed that is slow compared with the sampling rate.

#### With Singular G using Left Unimodular Matrix: Case 2

The requirement that the high-frequency gain matrix G be nonsingular is identical to the condition required for system

decoupling using state feedback.<sup>4</sup> When the matrix G is singular, u(k) cannot be synthesized using Eq. (11). A way of solving this problem, presented in Ref. 3, was to postaugment both the plant and model with a unimodular polynomial matrix  $U_L(z)$ . Since H(z) is assumed to be nonsingular, a suitable  $U_L(z)$  can always be found so that  $U_L(z) \cdot Cadj(zI-A)B$  is row proper. (See Theorem 2.5.7 in Ref. 5.)  $U_L(z)$  should be such, however, that the same relation as given by inequality (7) holds between the augmented plant and model, but this is usually cumbersome to check.

Find, now, the proper augmenting system as

$$H_C(z) = U_L(z) \cdot L_C^{-1}(z)$$

$$= C_C(zI - A_C)^{-1} B_C + D_C$$
(14)

where

$$L_C(z) = \text{diag}[l_{C_i}(z); i=1,2,...,r]$$
 (15)

and  $l_{C_i}(z)$  is an arbitrary stable polynomial whose order should be as low as possible. Its lowest possible order is  $\partial_{ci}[U_L(z)]$ , the *i*th column degree of  $U_L(z)$ . At the least,  $L_C(z)$  can be selected as  $l_C(z)I_r$  where  $\partial l_C(z) = \partial_{\max} U_L(z)$ . It should be noted that the above system has no zeros because  $U_L(z)$  is a unimodular polynomial matrix, and thus always has a stable inverse.

If the plant described by Eqs. (1) and (2) is augmented with system (14) such that the pulse transfer matrix of the augmented plant becomes  $[U_L(z)L_C^{-1}(z)]H(z)$ , the following system is obtained<sup>3</sup>

$$x_A(k+1) = A_A x_A(k) + B_A u(k)$$
 (16)

$$\eta_A(k) = C_A x_A(k) \tag{17}$$

where

$$x_A(k) = [x_C(k), x(k)]^+, x_C^T(0) = [0,...,0]$$

$$A_{A} = \begin{bmatrix} A_{C} & B_{C}C \\ 0 & A \end{bmatrix}, B_{A} = \begin{bmatrix} 0 \\ B \end{bmatrix}$$

$$C_{A} = [C_{C}, D_{C}C], \eta_{A} \in R'$$
(18)

and + denotes the block transpose.

Similarly, augmenting the model given by Eqs. (4) and (5) with Eq. (14) gives

$$x_B(k+1) = A_B x_B(k) + B_B u_M(k)$$
 (19)

$$\eta_B(k) = C_B x_B(k) \tag{20}$$

where

$$x_B(k) = [x_D(k), x_M(k)]^+, x_D^T(0) = [0,...,0],$$

$$A_{B} = \begin{bmatrix} A_{C} & B_{C}C_{M} \\ 0 & A_{M} \end{bmatrix}, \quad B_{B} = \begin{bmatrix} 0 \\ B_{M} \end{bmatrix}$$

$$C_{B} = \begin{bmatrix} C_{C}, & D_{C}C_{M} \end{bmatrix}, \quad \eta_{B} \in \mathbb{R}^{r}$$
(21)

The controller can be designed for these two augmented systems using exactly the same procedure as was used to derive the control input Eq. (11). Therefore, the control input which makes the pulse transfer matrix from  $u_M(z)$  through  $\eta_a(z)$  coincide with that from  $u_M(z)$  to  $\eta_b(z)$  is given by

$$u(k) = G_A^{-1} [F_A x_A(k) + q_A(k)]$$

$$q_A(k) = F_B x_B(k) + G_B u_M(k)$$
(22)

<sup>¶</sup>A full row rank matrix is called row proper if its coefficient matrix of highest-row-degree terms has full row rank.<sup>5</sup>

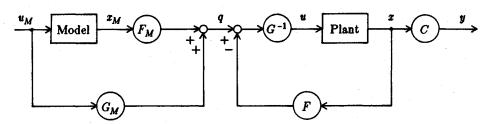


Fig. 1 The model-following controller with basic control law: Case 1.

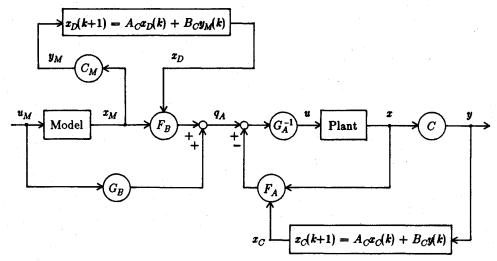


Fig. 2 The model-following controller with basic control law: Case 2.

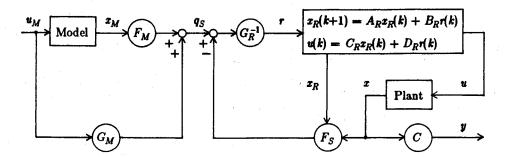


Fig. 3 The model-following controller with basic control law: Case 3.

where  $F_A$ ,  $G_A$  and  $F_B$ ,  $G_B$  are defined for the augmented plant and model in the same way as for F, G and  $F_M$ ,  $G_M$ . It should be noted that the high-frequency gain matrix  $G_A$  for the augmented plant is always nonsingular. The row-relative degrees may be larger than those for the original plant due to the fact that  $U_L(z)$  decreases the row degrees of the numerator and  $L_C(z)$  increases the denominator row degrees of the pulse transfer matrix.

It can be shown<sup>3</sup> that the control input equation (22) makes the pulse transfer matrix from  $u_M(z)$  to y(z) equal to the one desired. The controller structure for this case is shown in Fig. 2.

The modified control law can be obtained, in the same manner that was used to derive Eq. (12), as

$$u(k) = G_A^{-1} [-E_A x_A (k-1) - N_A U(k-1) + q_A (k)]$$

$$q_A(k) = E_B x_B (k-1) + N_B u_M (k-1) + G_B u_M (k)$$
 (23)

where

**JULY-AUGUST 1986** 

$$E_A = F_A A_A$$
,  $N_A = F_A B_A$ ,  $E_B = F_B A_B$ ,  $N_B = F_B B_B$  (24)

## With Singular G Using Right Unimodular Matrix: Case 3

An alternate method of providing model-following control for systems whose high-frequency gain matrix is singular, by post-augmenting the plant using a unimodular polynominal matrix  $U_R(z)$ , is described here. Before this is done, it should be recalled that 1) the orders of the plant and model can be different as long as condition (7) is satisfied, and 2) the pole-zero cancellation of the plant occurs in the feedback part, making the pulse transfer matrix of this portion  $\Phi^{-1}(z)$ , which can be specified arbitrarily by the designer as long as it is stable.

It is guaranteed by Theorem 2.5.14 in Ref. 5 that one can always find a suitable unimodular polynomial matrix  $U_R(z)$  such that  $[Cadj(zI-A)B] \cdot U_R(z)$  is row proper. Define, then, the proper augmenting system

$$H_R(z) = U_R(z) \cdot L_R^{-1}(z)$$
 (25)

where

$$L_R(z) = \text{diag}[l_{Ri}(z); \qquad i = 1, 2, ..., r]$$
 (26)

and  $l_{Ri}(z)$  is an arbitrary stable polynomial that is selected in the same manner as  $l_{Ci}(z)$ . Performing now the minimal

(39)

realization of Eq. (25) as

$$x_R(k+1) = A_R x_R(k) + B_R r(k), \quad x_R(0) = 0$$
 (27)

$$u(k) = C_R x_R(k) + D_R r(k)$$
(28)

where  $r(k) \in \mathbb{R}^r$  is to be determined later, the augmented plant  $H(z)H_R(z)$  can be expressed as

$$x_S(k+1) = A_S x_S(k) + B_S r(k)$$
 (29)

$$y(k) = C_S x_S(k) \tag{30}$$

where

$$A_{S} = \begin{bmatrix} A_{R} & 0 \\ BC_{R} & A \end{bmatrix}, B_{S} = \begin{bmatrix} B_{R} \\ BD_{R} \end{bmatrix}$$

$$C_{S} = \begin{bmatrix} 0 & C \end{bmatrix}, x_{S}(k) = [x_{R}(k) & x(k)]^{+}$$
(31)

Let r(k) be defined for the feedback part, using the same technique as described before, as

$$r(k) = G_S^{-1} [-F_S x_S(k) + q_S(k)]$$
 (32)

where  $G_S$  and  $F_S$  are defined for the augmented plant in the same way as G and F for the original plant, and where  $q_S(k)$  will be determined later. With this r(k), the pulse transfer matrix from  $q_S(z)$  through y(k) is expressed<sup>6</sup> as  $\Phi_S^{-1}(z)$  where  $\Phi_S(z)$  is defined for the augmented plant as in  $\Phi(z)$  by Eq. (10) for the original plant.

So that the pulse transfer matrix of the plant-controller combination can match that of the model, the precompensator part [from  $u_M(k)$  to  $q_S(k)$ ] must be  $\Phi_S(z)H_M(z)$ . Noting the row-relative degree observation one) described earlier, we select among its family a reference model which satisfies the condition that each row-relative-degree of  $H_M(z)$  is larger than or equal to the corresponding degree of  $H(z)H_R(z)$ . It can be seen now that  $q_S(k)$  should be defined, with the same design parameters used in the feedback controller design, as

$$q_S(k) = F_M x_M(k) + G_M u_M(k)$$
(33)

The pulse transfer matrix from  $u_M(k)$  to  $q_S(k)$  is calculated<sup>6</sup> as  $\Phi_S(z)H_M(z)$ , which makes the overall pulse transfer matrix the desired  $H_M(z)$ . Equations (27), (28), (32), and (33) determine the controller structure that is shown in Fig. 3.

The modified control law for synchronizing the control and measurement is obtained as

$$r(k) = G_S^{-1} [-E_S x_S(k-1) - N_S u(k-1) + q_S(k)]$$

$$q_S(k) = E_M x_M(k-1) + N_M u_M(k)(k-1) + G_M u_M(k)$$
(34)

where

$$E_S = F_S A_S$$
,  $N_S = F_S B_S$ ,  $E_M = F_M A_M$ ,  $N_M = F_M B_M$  (35)

## **Application to CCV Flight Control**

The design of flight control systems for CCV aircraft requires that they be considered as multi-input multi-output systems, because CCV modes are achievable using multiple control inputs and, usually, more than one output. In the following, a CCV flight controller is designed for the vertical translation ( $\alpha_2$ ) mode<sup>1,2</sup> which controls the vertical velocity at constant pitch attitude.

Using the stability coordinate axes and choosing the state variables as

$$x(t) = [\bar{u}(t), w(t), \theta(t), q(t)]^{T}$$
(36)

the dynamics of this mode are expressed as<sup>3</sup>

$$\dot{x}(t) = \begin{bmatrix} X_{\bar{u}} & X_{w} & -g & 0 \\ Z_{\bar{u}} & Z_{w} & 0 & U_{0} \\ 0 & 0 & 0 & 1 \\ \nabla_{\bar{u}} & \nabla_{w} & 0 & \Delta_{q} \end{bmatrix} x(t) + \begin{bmatrix} X_{\delta_{f}} & X_{\delta_{e}} \\ Z_{\delta_{f}} & Z_{\delta_{e}} \\ 0 & 0 \\ \nabla_{\delta_{f}} & \nabla_{\delta_{a}} \end{bmatrix} u(t)$$
(37)

where

$$\nabla_l = M_l + M_w Z_l, \qquad \Delta_g = M_g + M_w U_0 \tag{38}$$

In the above, l is either  $\bar{u}$ , w,  $\delta_f$ , or  $\delta_e$ ,  $\bar{u}(t)$  is the forward speed change, w(t) vertical speed change,  $\theta(t)$  pitch angle, q(t) pitch rate,  $\delta_f$  maneuvering flap deflection,  $\delta_e$  elevator defletion, and M, X, Z are stability and control derivatives.

This system is discretized at the sampling frequency of 40 Hz, giving

$$A = \begin{bmatrix} 1 & 0.001 - 0.004 & 0 \\ -0.002 & 0.978 & 0 & 0.108 \\ 0 & 1 & 1 & 0.025 \\ 0 & -0.011 & 0 & 0.970 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -0.029 - 0.051 \\ -0.001 - 0.007 \\ 0.057 & -0.543 \end{bmatrix}$$

If we select the output matrix C as

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{40}$$

and the design parameters  $f_i^j$  such that

$$f^{1}(z) = f^{2}(z) = z + 0.1 \tag{41}$$

it is found that

$$G = \begin{bmatrix} -0.029 & -0.051 \\ -0.057 & -0.543 \end{bmatrix}, F = \begin{bmatrix} -0.002 & 1.078 & 0 & 0.108 \\ 0 & -0.011 & 0 & 1.069 \end{bmatrix}$$
(42)

where  $m_1$  and  $m_2$  are 1.

The reference model selected is

$$A_{M} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.81 & 1.8 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.81 & 1.8 \end{bmatrix}, \qquad B_{M} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_{M} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \end{bmatrix}$$

$$(43)$$

which yields

$$G_M = 0_2, \quad F_M = \begin{bmatrix} 0.01 & 0.1 & 0 & 0 \\ 0 & 0 & 0.01 & 0.1 \end{bmatrix}$$
 (44)

The basic control law is thus given by

$$u(k) = \begin{bmatrix} 0.080 & -45.130 & 0 & -0.360 \\ -0.009 & 4.796 & 0 & -1.932 \end{bmatrix} x(k) + \begin{bmatrix} -0.418 & -4.184 & 0.039 & 0.389 \\ 0.044 & 0.443 & -0.023 & -0.225 \end{bmatrix} x_M(k)$$
(45)

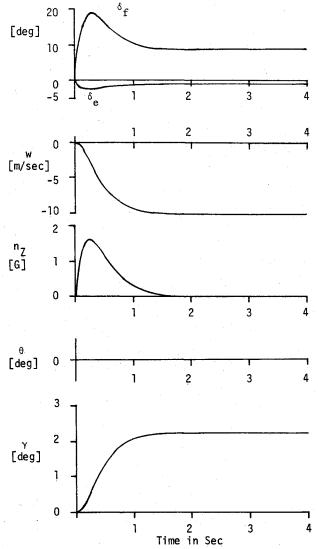


Fig. 4 Simulation results for vertical translation mode.

and the modified input by

$$u(k) = \begin{bmatrix} 0.167 & -44.208 & 340.856 & 6.521 \\ -0.019 & 4.765 & -197.326 & -8.156 \end{bmatrix} x(k-1)$$

$$+ \begin{bmatrix} 3.390 & -7.951 & -25.099 & 58.875 \\ -0.359 & 0.842 & 14.530 & -34.084 \end{bmatrix} x_M(k-1)$$

$$+ \begin{bmatrix} 0.895 & -1.731 \\ 0.228 & 3.241 \end{bmatrix} u(k-1)$$

$$+ \begin{bmatrix} -4.185 & 30.987 \\ 0.443 & -17.939 \end{bmatrix} u_M(k-1)$$
(46)

The computational result is shown in Fig. 4 for the reference input  $u_M(k) = [-1 \ 0]^T$  which commands the aircraft to reduce the vertical speed and to maintain pitch angle simultaneously. It shows that the vertical velocity is decreased by 10 m/s while the pitch angle is unchanged. This means that mode  $\alpha_2$  is achieved. The control input given by either Eq. (45) or (46) gives the same results in computer simulations.

Since the control method presented requires the plant to have minimum phase, care has to be exercised in selecting the sampling frequency. This is because unstable zeros can appear for some sampling frequencies, even though the zeros of the continuous-time system are stable.7 This dependence on the minimum phase property of the plant is not unique to the proposed method, but common to all control methods which rely on the plant pole-zero cancellation.

## Conclusions

A discrete-time model-following control method that uses a plant state vector was presented. Synthesizing the control input using the plant outputs, rather than the plant state vector, can be achieved using exactly the same method as presented previously<sup>3</sup> and, therefore, it was not described in this paper. When the plant state vector and parameters are simultaneously unknown, an adaptive observer, which provides the designer with these quantities, may be used.8

The modification of the control laws for synchronizing the sampling and control was described and shown by simulation study to be useful. This becomes more important when adaptive control is employed, since the control input computation time increases due to parameter estimate updating, which precedes the control input computation.8 Although the modified control law is mathematically equivalent to the basic law which is based on the modeled plant, it may cause a deterioration of the control performance because of the unmodeled dynamics and inaccurate parameters of the plant. However, since the plant is usually modeled such that the basic law performs well, it is expected that the modified control law works as well as the basic one.

The control input was determined such that the input to the stable difference equation of the output error is null. When this equation does not have a solution, that is, when the highfrequency gain matrix is singular, the plant is preaugmented using a unimodular polynomial matrix. Since there is no need for augmenting the model, the order increase in the controller is generally smaller than that required in the method presented previously.<sup>3</sup> In addition, the selection of the suitable reference model is easier with this method. It is guaranteed that a suitable unimodular polynomial matrix always exists such that the transfer matrix of the preaugmented plant is row proper as long as the plant transfer matrix is nonsingular, which is the usual case. The control inputs thus obtained are shown to achieve transfer matrix matching between the plant-controller combination and the model.

The model-following control system was found to be effective in achieving a control-configured vehicle model. The simulation study was carried out using the data for the T-2-CCV aircraft and showed that that design was effective. The design method can be applied for the lateral modes in the same manner.

## References

<sup>1</sup>Whitmoyer, R.A. and Ramage, J.K., "The Fighter Control Configured Vehicle (CCV) Program Development and Flight Test Summary," Proceedings of the 8th Annual Symposium, Society of Flight Test Engineers, Washington, DC, 1977.

<sup>2</sup>Ramage, J.K. and Swortzel, F.R., "Design Considerations for Implementing Integrated Mission-Tailored Flight Control Modes," AGARD-CP-257, 1978, pp. 16.1-16.8.

Kanai, K., Uchikado, S., Nikiforuk, P.N., and Hori, N., "Application of a New Multivariable Model Following Method to Decoupled Flight Control," Journal of Guidance, Control, and Dynamics, Vol. 8, Sept.-Oct. 1985, pp. 637-643.

<sup>4</sup>Falb, P.L. and Wolovich, W.A., "Decoupling in the Design and Synthesis of Multivariable Control Systems," IEEE Transactions on Automatic Control, Vol. AC-12-6, 1967, pp. 651-659.

<sup>5</sup>Wolovich, W.A., *Linear Multivariable Systems*, Springer-Verlag, New York, 1974.

<sup>6</sup>Kanai, K., Hori, N., and Nikiforuk, P.N., "A Discrete-Time Multivariable Model Following Method Applied to Decoupled Flight Control," Proceedings of the AIAA 12th Guidance, Navigation, and Control Conference, Snowmass, CO, Aug. 1985, pp.

Astrom, K.J., Hagander, P., and Sternby, J., "Zeros of Sampled Systems," Automatica, Vol. 20, 1984, pp. 31-38.

Hori, N., Ph.D. Dissertation, University of Saskatchewan, Saskatoon, Canada, April 1985.